Tutorial 7:

3.2.6. t is Feb 1st 2004. $I_t = 103$, $I_{t+1} = 98$, $i = 0.08$

(a) $PR_t = K - I_t = K - 103$, $PR_{t+1} = K - I_{t+1} = K - 98$.

Since $\frac{PR_{t+1}}{PR_t} = 1+i = 1+8\%$ then $(K-98) = 1.08(K-103) \Rightarrow K = 165.50$

$PR_{t+1} = 165.50 - 98 = 67.50$

(b) Set n as the duration of repayment (start from Feb, 2003)

(t as Feb 2004, t-1 as Feb 2003)

$OB_{t-1} = \frac{I_t}{i} = \frac{103}{0.06} = 1287.50$.

Set OB_{t-1} as initial value, then

$OB_{t-1} = K\, a_{\overline{n}|0.08} \Rightarrow 1287.50 = 165.50\, a_{\overline{n}|0.08} \Rightarrow n = 12.7$

(165.50)

which means 12 regular payments and a smaller final payment X,

Then $1287.50 = 165.50\, a_{\overline{12}|i} + X \cdot v_i^{13} \Rightarrow X = 109.54$

The date of final payment is Feb, 2016.

3.2.9. Suppose the payment bank A per month.

(1 per month.)

$i = \frac{i^{(12)}}{12} = 0.01$.

The repayment starts at the end of 5 yr.

$OB_{5\times12} = OB_{60} = 1 \cdot a_{\overline{180-60}|0.01} = a_{\overline{120}|0.01} = 69.7$

Plus the penalty, $L = OB_{60} \times (1+k\%) = 69.7(1+k\%)$ is loan from Bank B.

Assume repayment is R, consider the time point at the end of 5th year

$R \cdot a_{\overline{120}|0.0075} = L \Rightarrow R = 0.883 \cdot (1+k\%)$.

correct it $R \le 1 \Rightarrow 0.883(1+k\%) \le 1 \Rightarrow k \le 0.1326$

7.2.12.

At $i^{(12)} = 0.12$. the monthly payment $k = \frac{100,000}{a_{\overline{25}|0.01}} = 1053.22$.

after 3 years. Outstanding balance $OB_{12\times3} = 1053.22\, a_{\overline{300-36}|0.01} = 97,707.45$.

Jones still pay the same amount of money.

~~$OB_{12\times3}$~~.

$97707.45\, v^{12}_{0.1035} + 1053.22\, a_{\overline{36}|0.0275} = 92,858.$

Smith receives 192,858 in total.

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① When $k_1 = k_2 = \cdots = k_n$

$OB_t = OB_0(1+i)^t - k s_{\overline{t}|i}$ (retrospective)

$= k\, a_{\overline{n-t}|i}$ (prospective)

② $I_t = OB_t \times i = k a_{\overline{n-t}|i} \times i = k(1 - v^{n-t+1})$

③ $PR_t = k - I_t = k v^{n-t+1}$

$\frac{PR_t}{PR_s} = \frac{k v^{n-t+1}}{k v^{n-s+1}} = v^{s-t} = (1+i)^{t-s}$